Physics I
ISI B.Math
Final Exam : May 13, 2013

Total Marks: 50

Answer question 1 and any 4 from the rest

1. **(Marks :** $2 \times 5 = 10$**)** Mark one correct answer.

(a) Consider a pendulum of length l with a bob of mass m making an angle θ with the vertical. Which of the following is NOT a valid Lagrangian for the system giving the correct equations of motion.

(i) $L = \frac{1}{2}ml^2\dot{\theta}^2 - mgl(1-\cos\theta) - mgl\sin\sqrt{\frac{g}{l}}t$
(ii)$L = \frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta + m\sqrt{gl^3}\cos\theta\dot{\theta}$
(iii)$L = \frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta + mglt\sin\theta\dot{\theta}$
(iv)$L = \frac{1}{2}ml^2\dot{\theta}^2 + mglt\sin\theta\dot{\theta}$

(b) Which of the following statements is false?

(i) The Lagrangian of a system is in general not invariant under a Galilean transformation (i.e, transformation to another inertial frame)
(ii) The Lagrangian of a system is invariant under a Galilean transformation.
(iii) The Euler-Lagrange equations for a system are invariant under a Galilean transformation
(iv) The total energy of a system is not invariant under a Galilean transformation.

(c) A particle of mass m is moving under the influence of a force $\mathbf{F} = -k\mathbf{r}$, where k is a positive constant. Which of the following statements about its motion is not true?

(i) Angular momentum about the origin is conserved.
(ii) The orbits can be bounded or unbounded depending on the total energy.
(iii) The particle obeys Kepler's 2nd Law.
(iv) The Lagrangian of the particle is invariant under rotations about any axis through the origin.

(d) A tetherball of mass m is attached to a post of radius R by a string. Initially it is at a distance r_0 from the center of the post and is moving tangentially with a speed v_0. The string passes through a hole at the center of the post at the top.The string is gradually shortened by drawing it through the hole. Ignore gravity. Until the ball hits the post

(i) The energy and the angular momentum about the center of the post are constant.
(ii) The energy of the ball is constant but the angular momentum about the center of the post changes
(iii) Both the energy and angular momentum change
(iv) The energy of the ball changes, but the angular momentum about the center of the post is constant

(e) Newton's Third Law does not hold for the following system:

(i) Two static electrons separated by a distance r.
(ii) Two protons moving with uniform velocities along the x and y directions respectively.
(iii) A binary star system of two stars in orbit around each other under the influence of their mutual gravitational interaction.
(iv) An inelastic collision between two balls

2. **(Marks : 10)**

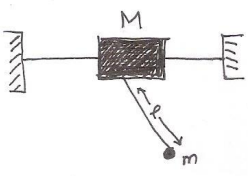
A mass m on a spring with spring constant k oscillates horizontally on a frictionless table. The amplitude is d. At the moment ($t = 0$) when the mass is at the position $x = d/2$(and moving to the right), it collides and sticks to another mass m which was sitting on the table and the two start moving together. What is the resulting $x(t)$? What is the amplitude of the new oscillation ? Why is it smaller than the amplitude before collision ?

3. **(Marks : 10)**

A solid sphere of mass M and radius R rotates freely in space with an angular velocity ω about a fixed diameter. A particle of mass m, initially at one pole , moves with constant speed v along a great circle of the sphere. When the particle has reached the other pole after a time T, the rotation of the sphere would have been retarded by an angle α compared to the rotation of the sphere without the particle moving on its surface. Find α in terms of ω, T, M and m. (The moment of inertia of a solid sphere of mass M and radius R about its diameter is $\frac{2}{5}MR^2$).

4. **(Marks: 5 + 5)**
In an elastic collision(scattering) between an electron with kinetic energy E and an electron at rest, the incoming electron is observed to be deflected through an angle of $30°$.
(a) What are the energies of the two electrons after the collision ?
(b) Make a sketch of the above collision in the centre of mass (Zero momentum) frame. Label the sketch with the appropriate values of the scattering angle and initial and final velocities of the target and incoming electrons.

5. **(Marks: 4 + 2 + 4)**



A mass M is free to slide along a frictionless rail. A pendulum of length l and mass m hangs from M.
(a) Write down the Lagrangian for the system and hence find the equations of motion.
(b) Identify the cyclic coordinate and show that the corresponding generalized momentum is conserved.
(c) Find the general solution of the equation of motion in the small oscillation approximation.

6. **(Marks: 6 + 4)**

(a) Two particles with mass m_1 and m_2 interact with gravitational forces. They start out from rest a distance ρ apart and are allowed to fall into each other. How long does it take for them to collide ?
(You may need to use the following result : $\int_0^1 \frac{dx}{\sqrt{\frac{1}{x}-1}} = \frac{\pi}{2}$)

(b) A particle moves in a circular orbit in a force field given by

$$F(r) = -\frac{k}{r^2}$$

Show that if k suddenly decreases to half its original value, the particle's orbit becomes parabolic.